

Linear Algebra I: MATH-GA 2110 Fall 2014
Tentative Syllabus

- Week 1:** Vector space axioms and their consequences; finite fields \mathbb{Z}_p , examples of vector spaces; polynomial rings $\mathbb{F}[x]$, multi-index notation and polynomials $\mathbb{F}[x_1, \dots, x_n]$ in several variables; proving the Degree Formula for $\mathbb{F}[x_1, \dots, x_n]$; (abstract polynomials) \neq (polynomial functions) for finite fields; Fermat's Little theorem.
- Week 2:** Linear systems of equation; vector subspaces and examples; matrix equations $AX = B$; solution via row/column operations; determining the linear span of vectors; a computational case study; homogeneous vs inhomogeneous equations.
- Week 3:** Spanning sets, independent sets, and bases; definition of $\dim_{\mathbb{F}}(V)$; "parametric" and "basis" descriptions of a subspace; various methods for finding subspace bases; computational examples; Lagrange Interpolation as a linear algebra problem.
- Week 4:** Quotient spaces V/W ; algebraic structure in V/W ; Dimension Theorem for quotients; constructing bases in V/W ; computational examples. Linear operators; it rank, *nullity* and the Dimension Theorem; computational examples.
- Week 5:** Finding bases for $\text{Range}(T)$ and $\text{Ker}(T)$. Decomposition of operators: invariant subspaces and the induced maps $T|_W : W \rightarrow W$, $\tilde{T} : V/W \rightarrow V/W$; block upper-triangular form; the First Isomorphism theorem. Direct sum decompositions $V = V_1 \oplus \dots \oplus V_r$; examples and a Dimension Formula; block-diagonal form; direct sums and associated projection operators $P_i : V \rightarrow V$; oblique projections vs orthogonal projections; projections and idempotent operators $P^2 = P$.
- Week 6:** Computational examples of projections. Eigenspaces, eigenvalues and spectrum $\text{sp}(T)$; idagonalizable operators; span of the eigenspaces is a direct sum $W = \bigoplus_{\lambda \in \text{sp}(T)}$. Matrices vs operators – algebraic properties and examples; covariance of matrix descriptions $[T]_{\mathfrak{H}\mathfrak{K}}$.
- Week 7:** Change of basis; similarity transforms; similarity classes as orbits; computational examples. The dual space V^* ; dual bases and computed examples. Transpose $T^t : W^* \rightarrow V^*$ of $T : V \rightarrow W$; algebraic properties; matrix descriptions of T^* vs T ; computational examples.
- Week 8:** Reflexive spaces and $(T^t)^t$ vs T ; annihilators W° and the Dimension Theorem; Theorem: row rank = column rank. Permutations and the symmetric group S_n ; disjoint cycle decompositions; parity $\text{sgn}(\sigma)$ is well-defined; the determinant $\det(A)$; some algebraic properites; computed examples.
- Week 9:** Row/column operations and determinants; elementary operations and their description by elementary matrices; Application: A^{-1} exists $\Leftrightarrow \det(A) \neq 0$; Application: multiplicative property of $\det(A)$; (brief) Cramer's rule and expansion by minors; computational efficiency of various methods.

- Week 10:** Eigenvalues and the characteristic polynomial p_T, p_A of a linear operator or matrix; Fundamental Theorem of algebra and factorization of p_T ; (brief) the quadratic equation. Eigenvalues and eigenspaces $E_\lambda(T)$; geometric vs algebraic multiplicities; diagonalization and computational examples; failure to diagonalize.
- Week 11:** Convergence in matrix space $\|A_n \rightarrow A\|_\infty$; Cauchy convergence criterion and convergence of the exponential series $e^A = \sum_{n=0}^\infty A^n/n!$; the Exponent Law $e^{A+B} = e^A \cdot e^B$ for commuting matrices. Application: computing e^A for a diagonalizable matrix. Application: solving linear system of ODE: $D_t x(t) = A \cdot x(t)$ with initial condition.
- Week 12:** Inner product spaces over \mathbb{R} and \mathbb{C} ; Schwartz inequality, triangle inequality; examples. Orthonormal sets and bases; Bessel's inequality; orthogonal complements in finite dimensional case; orthogonal projections and orthogonal direct sum decompositions; the Gram-Schmidt construction of ON bases; Legendre polynomials; a Fourier series example.
- Week 13:** Orthogonal vs ordinary diagonalizability; Schur normal form. The adjoint operator T^* and its properties; self-adjoint projections and orthogonal decompositions of V ; self-adjoint, unitary, and normal operators ($T^*T = TT^*$) on an inner product space; orthogonal diagonalizability $\Leftrightarrow T$ is normal; unitary operators and change of orthonormal basis. Rigid motions on \mathbb{R}^n ; reflections; rotations and Euler's theorem for \mathbb{R}^3
- Week 14:** The spectral theorem for vector spaces and inner product spaces; functions $f(T)$ of diagonalizable operators; positive definite operators and the polar decomposition (singular value decomposition). Summary review.

G63.2110 Linear Algebra I, Fall 2014

Course Information

Textbooks

Linear Algebra, by Friedberg, Insel, Spence, 4th Edition

Schaum's Outline Series: Linear Algebra, by Seymour Lipschutz.

Much of the course will be based on Prof. Greenleaf's printed *Class Notes* which will be distributed in class in hardcopy form, with F/I/S as a backup. The *Class Notes* will also be posted on the NYU Courses site as pdf files. F/I/S will be a source of problem assignments (copies of those will be distributed for those who do not have a current edition of F/I/S). This course presumes familiarity with much of the material in pp.1-100 of the Schaum's Outline text, especially row/column operations, Gaussian elimination, matrix multiplication and inversion.

Exams and Grading Policy

There will be about four Problem Sets, each including a subset of problems to be handed in for grading. These hand-in problems will count for about one-third of your grade. The other two-third will be based on a Takehome Final and an In-Class Final carrying equal weight toward your final grade; both will be comprehensive in their coverage of topics from the term. The In-Class exam will be "open-book/ open-notes" – this exam is a test of understanding rather than memory, but obviously if you have not mastered the main topics by doing problems on your own, an open book will not be much help. The In-Class exam will focus on your grasp of general course content and particularly important computational skills, with most proofs restricted to the Takehome Exam.

No Internet-capable electronic devices will be permitted during the In-Class exam – no laptops, no i-phones, no asking Siri for answers. *Class Notes* will be distributed in hardcopy form and can be brought to the exam. If there is other material you want to use loaded on your laptop, you will have to make hardcopies and bring to the exam.

Regarding Your Final Grade

Your tentative term grade will be determined as described above, subject to the following condition:

If there is an enormous disparity between your grade on the In-Class and Takehome exams – for instance a 99 on the Takehome and 25 on an In-Class exam for which the median grade was well north of 50 – this raises disturbing questions about the validity of all your classwork grades. Therefore in such cases I reserve the right to withhold issuing any final grade until you take a supplementary oral exam in my presence, in which you will be asked to reconstruct some of your answers from the Takehome Final, without referring to your exam paper.

For this reason, copying someone else's work and presenting it as your own on the Takehome portion of the Final Exam, will carry substantial risks.

The Problem Sets

In each Problem Set, full handwritten solution to all problems marked by (*) are included in the hardcopy Problem Set packet distributed in class. (The questions, but not the handwritten solutions to the (*) problems, will be posted on NYU Courses.) Problems whose numbers are “boxed,” as in 5, are the Hand-In problems for that set. Full solutions to the hand-in problems will be distributed when all papers have been turned in. Due dates for the Hand-In sets will be posted on NYU Courses.

The calendar for Fall 2014 final exams has already been posted on the NYU Home/Registrar web-site. The In-Class exam will be held at that time, with extra time allowed if possible; the Takehome Final will be distributed about two weeks before the end of the term and will be turned in one or two days before the time of the In-Class exam (so we can get started on grading), precise due date to be announced.

Contact Information/Office Hours

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Tentative office hours for the Fall 2014 term are

4:00-5:00 Tuesday and 11:00-12:30 Thursday

or by appointment. In general, if I am in the office and not infernally busy I am generally willing to answer questions on the spot. I am likely to be in the office most mornings 11:00-1:00.